RESEARCH ARTICLE

Invariant spatial information in sketch maps — a study of survey sketch maps of urban areas

Jia Wang[1,2] and Angela Schwering[2]

[1]Department of Mathematical Sciences, University of Greenwich, UK
[2]Institute for Geoinformatics, University of Münster, Germany

Abstract: It is commonly recognized that free-hand sketch maps are influenced by cognitive impacts and therefore sketch maps are incomplete, distorted, and schematized. This makes it difficult to achieve a one-to-one alignment between a sketch map and its corresponding geo-referenced metric map. Nevertheless, sketch maps are still useful to communicate spatial knowledge, indicating that sketch maps contain certain spatial information that is robust to cognitive impacts. In existing studies, sketch maps are used frequently to measure cognitive maps. However, little work has been done on invariant spatial information in sketch maps, which is the information of spatial configurations representing correctly the real world. We aim to study such information from a cognitive perspective. This paper first presents basic spatial objects identified in sketch maps and then introduces sketch aspects that capture invariant spatial information. The accuracy and reliability of these aspects were evaluated by a human study. We collected sketch maps from participants, extracted and measured spatial relations of identified spatial objects, and in the end analyzed the accuracy and statistical significance of these relations. Based on the statistical survey, we propose in this paper a set of seven sketch aspects that constitute invariant spatial information, along with a spatial analysis method to measure them. The findings of these aspects help to understand which spatial information is preserved under the transformation from the physical world to human sketch maps.

Keywords: sketch map, sketch aspect, invariant spatial information, cognitive impact

1   Introduction

Sketch maps in history have long been used to recall, visualize, and communicate spatial knowledge about spatial scenes. People have associations with the environment they live in or have visited. People can have direct access to their surrounding spaces from what they perceive, experience, and memorize. Most people are able to draw maps to convey their spatial knowledge (e.g., [19]). These sketch maps are usually incomplete, distorted, and schematized due to cognitive impacts [9, 29, 31]. In spatial cognition, sketch maps have been used to measure cognitive maps (or mental maps). There have been plenty of studies focusing on inaccuracy or errors in spatial knowledge, which originate from cognitive maps reflected in sketch maps. For example, geometric properties such as angles are usually rectangularized [4]. Spatial relations, such as distances, are judged differently between locations due to the effect that a location is judged closer to a reference-point, like a landmark, than vice versa [15]. However, so far hardly any studies have tackled the spatial information that is preserved in sketch maps, termed *invariant spatial information* in this paper. Existing spatial analysis methods developed for conventional metric maps are not applicable to sketch maps, because we require an analysis that captures solely the spatial aspects preserved in sketch maps. These issues all point to the need for a new study that investigates the invariant spatial information in sketch maps and develops a corresponding spatial analysis method.

In this paper, we propose a set of sketch aspects of invariant spatial information, along with a new method to represent and measure these aspects. Sketch maps studied in this paper reflect map producers' survey knowledge about urban environment. Each sketch aspect represents one qualitative spatial relation that is preserved from the physical world to a sketch map via cognitive mapping. These sketch aspects fulfill the following two requirements: they are (1) cognitively adequate and (2) accurate and reliable. Cognitively adequacy requires empirical evidence to support our claim that the proposed sketch aspects conform with what people are able to recall and draw on their sketch maps. The choices of qualitative spatial relations and distinctions are justified by perceptions and understandings of spatial configurations through experiments. Accuracy and reliability require that the proposed sketch aspects are consistently accurate in comparison with metric maps across varied study areas and sketch map producers.

This study contributes to spatial cognition research by revealing the invariant spatial information that humans are able to perceive from reality, retrieve from memory, and draw correctly on their sketch maps. The outcome of this study also lends cognitive support to the Sketchmapia project, which aims to develop a sketching interface and allows users to query spatial databases by free-hand sketch maps [26].

The remainder of the paper is organized as follows. We start in Section 2 with a review of related sketch map studies. We continue in Section 3 by explaining our approach to decide, represent, and measure invariant spatial information in sketch maps. We report in Section 4 an experiment conducted for validating accuracy and reliability of the sketch aspects proposed in Section 3. We end in Section 5 with conclusions and future work.

2   Background

Previous work has already investigated the characteristics and basic components of sketch maps and developed different approaches to sketch map analysis, upon which our study

is based. This section briefly reviews this background, explains how the related work is adapted in this paper, and what research gaps remain.

A cognitive map is a mental model that encompasses the internal process which enable people to acquire and use information about physical environments [9]. Information in cognitive maps is not as it is in two-dimensional cartographic maps. "Instead, cognitive maps are complex, highly selective, abstract, generalized representations in various forms" [9, p. 18]. Sketch maps, as the externalization of cognitive maps, reflect distortions, abstractions, and schematizations that originate in cognitive maps. Typical distortions include: distances between near spatial objects are considered relatively farther than distances between more distal ones [15]; ordinary buildings are judged closer to landmarks than the other way around [20]; routes are judged longer with more turns and intersections [23, 24] or more clutter (such as intervening cities [27]). Spatial information is also simplified in cognitive maps. For instance, angles tend to be remembered more rectangular [4], and curved features are recalled straighter [21, 30]. Cognitive impacts also result in errors, which are reflected in sketch maps as errors of quantity, shape, size, and inconsistent scales. However, there must be certain information conveyed correctly in sketch maps as it is understandable for people despite these cognitive impacts. Regarding the characteristics of sketch maps and the fact that they can be used to communicate spatial information, there are two principles being followed in this paper: first, sketch maps necessarily contain invariant spatial information (originated in cognitive maps) in order for people to be able to use them in the physical environment; second, cognitive impacts should be taken into account when sketch maps are under analysis because they cause distortions and schematizations in sketch maps.

Lynch [19] proposed five key elements found in the sketch maps of American cities. He names these elements as paths, edges, districts, nodes, and landmarks. Other researchers confirmed his finding with variations in the importance of the five elements in different types of urban environments, cf. [1,8,11,14]. Following Lynch, we distinguish in this paper four types of spatial objects as basic elements in sketch maps, namely landmarks, street segments, junctions, and city blocks (detailed descriptions in Section 3.1).

Both quantitative and qualitative methods have been applied to analyze sketch maps, cf. [1, 2, 10–12, 16, 32]. One of the early and notable works, "spatial-query-by-sketch" from Egenhofer [10], was founded on a mathematical model of spatial relations and their relaxations. Egenhofer suggested a sketch representation by using five types of spatial relations. These were the coarse topological relations represented by the 9-intersection model; the detailed topological relations "expressed by the component invariant table for non-empty boundary-boundary sequences" [10, p. 410]; the metric refinements of topological relations; the coarse projection-based cardinal directions partitioning the space into nine regions; and the detailed cardinal directions described by the percentage that a sketched object extends over the nine different regions. Our method of sketch map analysis is initially inspired by spatial-query-by-sketch. We propose seven sketch aspects of invariant spatial information (detailed descriptions in Section 3.3).

3   Sketch aspects and spatial analysis method

This section introduces a set of seven sketch aspects and explains how they are developed. The section later describes for each sketch aspect how it is spatially represented and measured.

3.1   Spatial objects in sketch maps

We define four types of spatial objects: landmarks, street segments, junctions, and city blocks upon which sketch aspects can be further built. *Landmarks* are non-path-like atomic elements. Unlike Lynch who defined landmarks as point features [19], we allow landmarks to be point or areal features depending on how landmarks are drawn and which method is used for representation and measurement. *Street segments* are also atomic elements. A street segment can either be a segmentation of a main street between junctions or a branching street connecting to a main street by a junction. Most sketched street segments represent public city streets whereas the remaining ones represent private and personal paths such as short-cuts or bike-paths. Connected street segments form "paths" as defined by Lynch as they are channels along which people can travel [19]. Street segments may also be part of the "edges" from Lynch [19]. *Junctions* are atomic elements as well. A junction is a place where it meets at least one street segment. Junctions belong to the "nodes" type defined by Lynch [19], and they are usually treated as point features in our study. Street segments and junctions form street networks. Street networks represent the street system that forms the urban transportation network in the physical environment. They are similar to the "path networks" from Lynch [19] in that a path network contains a main road, junction angles, and branchings. *City blocks* are the smallest two-dimensional areas formed by street segments intersecting at junctions. They are super elements composed of street segments and junctions and correspond to the "districts" defined by Lynch [19]. City blocks provide space to locate landmarks. Figure 1 shows an example of the four spatial objects defined in this paper. We label in the figure two T-shape junctions, their shared street segment, a café as a landmark, and a city block containing a library.
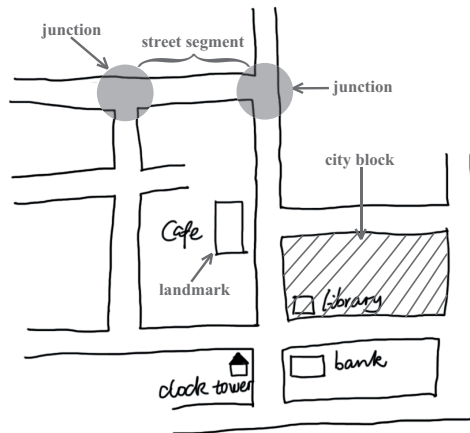

Figure 1: Sample spatial objects defined in a sketch map.

3.2   Development of sketch aspects

In a series of human-subject experiments [25, 33, 34, 36, 37], we asked participants to draw maps of urban areas they were familiar with. We then evaluated accuracy of sketch maps by comparing them with corresponding metric maps at both spatial object and spatial scene level (Table 1).

Levels of evaluation	Types of evaluation		
Spatial object level	Geometric attributes	Types Shapes Sizes	
Spatial scene level	Spatial relations	Quantitative	Distances Angles
		Qualitative	Topology Orientation Order

Table 1: Levels and types of evaluations.

The evaluation results showed that the qualitative spatial relations at the spatial scene level provided the best results. We discuss the major findings below, including briefly reporting on unsuccessful aspects that failed to capture invariant sketch information to give a complete picture of developing sketch aspects.

At the spatial object level, we found geometric attributes distorted and schematized. Street networks extracted from sketch maps were incomplete and simplified. Less than 20% of the street segments in study areas were drawn. Street segments were aggregated when participants ignored or forgot to draw connected junctions. Due to aggregation, city blocks formed by exact street segments could hardly be found. Participants recalled and drew far fewer landmarks than junctions and street segments, and they drew landmarks either as blobs or rectangles rather than actual shapes in reality or footprints in metric maps. Thus, geometric attributes at spatial object level cannot be considered as invariant spatial information.

At the spatial scene level, we also found quantitative spatial relations distorted (distance and angle). Distance relations (between landmarks and street segments, between landmarks and junctions) were found either lengthened or shortened without discernable patterns. There was no evidence that relative distance relations were in proportion to the sizes of landmarks or lengths of street segments. We also found rectangularized angles of connected street segments. Therefore, neither relative distances nor angles can be considered as invariant spatial information in sketch maps. We assessed the qualitative spatial relations as proposed by Egenhofer's spatial-query-by-sketch [10]: five types of spatial relations of topology and cardinal directions including metric refinements were analyzed. Low accuracy was found in comparison with metric maps, e.g., the accuracy rate of topological relations and projection-based cardinal directions of landmarks were both lower than 60% [33].

Inspired by Egenhofer's work [10] and following the suggestions from our previous empirical studies [25, 33, 34, 36, 37], we propose seven new sketch aspects (Table 2). Theoretically, these new sketch aspects realize spatial contiguity, association, connectivity, and spatial structure in sketch maps [13], which have been acknowledged to be able to capture the structural essence of spatial scenes in spatial similarity studies [3, 18]. Practically, the

decision of these aspects is derived from our empirical findings of sketch map accuracy: these aspects are correlated with human spatial thinking and consistently show high accuracy (greater than 90%) in the alignment of sketched scenes and their counterparts from metric maps [34].

Reference spatial objects ⟍ Referent spatial objects	Street segments	Junctions	City blocks
Street segments	Topology (1) Orientation (2)		
Landmarks	Orientation (3)		Topology (6)
Landmarks & Street segments	Linear order (5)	Cyclic order (4)	
City blocks			Topology (7)

Table 2: The new seven sketch aspects numbering from (1) to (7).

3.3   Sketch aspects and spatial analysis method

Regarding the fact that sketch maps have inconsistent scales and contain distortions, we introduce two analytical rules.

Rule 1:   We analyze different sketch aspects at different levels: local or global. Sketch aspects (1) topology of street segments, (6) topology of landmarks and city blocks, and (7) topology of city blocks are globally computed. Due to schematizations, the overall layout of the sketch map is distorted. Therefore, the remaining sketch aspects (order and orientation) are computed at local level. For example, we compute the orientation of a landmark only with respect to its adjacent but not distant street segments (sketch aspect (3)). Orientation relations between distant street segments and landmarks are not reliable because people draw distorted (e.g., straightened) streets in-between distant landmarks and street segments, and this way distorts also the orientation relations. We distinguish four types of *adjacency*: adjacency of landmarks and street segments, adjacency of junctions and landmarks, adjacency of landmarks and routes, and adjacency of street segments and routes. Their definitions are introduced together with the seven sketch aspects as below.

Rule 2:   Following Tversky's statement that "in cognition, elements are represented relative to each other and relative to a spatial reference frame" [28, p. 26], we argue that spatial objects themselves should serve as reference objects to locate one another. For locally computed sketch aspects, the positions of sketched spatial objects should use relative reference frames based on other sketched spatial objects, rather than the absolute reference frames such as cardinal direction systems.

Sketch aspect 1.   *Topology of street segments* describes the connectivity of street segments in a street network. It is computed at global level. The street network extracted from a sketch map can be analyzed as a graph: each street segment is represented as an edge

defined by two distinct nodes as junctions (Figure 2). For special cases, such as a "dead-end" street segment or a street segment ending at the boundary of a sketch map, one of its nodes is not connected to any other street segments. Let S be a set of street segments $S = \{s_1, s_2, \ldots, s_n\}$ each of which is definable by a pair of junctions. Then we define topology of street segments as in Equation 1:

$$TopologyS(s_1, s_2) = \begin{cases} \text{connected} & \text{if } s_1 \text{ and } s_2 \text{ share at least one junction} \\ \text{disconnected} & \text{otherwise} \end{cases} \qquad (1)$$
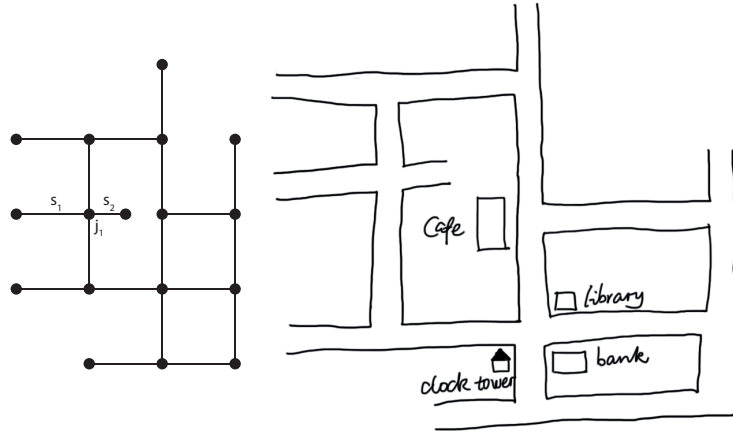

Figure 2: Street network (left) extracted from a new sketch map (right).

In Figure 2, street segment s_1 is connected to street segment s_2 at junction j_1. Street segment s_2 is an example of a dead-end street, and street segment s_1 is an example of a street ending at the boundary of a sketch map.

Sketch aspect 2. *Orientation of street segments* describes the binary directional relations between connected street segments. It is computed locally, i.e., only between connected streets. Let street segment s_1 be the reference object with its orientation pointing from its start to its end junction. Street segment s_1 and its connected street segment s_2 determine six directional relations: {front, front-left, front-right, back, back-left, back-right}. Let $\angle s_1 s_2$ be the angle between s_1 and s_2, we define the measure of the six directional relations in Equation 2:

$$OrientationS(s_2, s_1) = \begin{cases} \text{front} & \angle s_2 s_1 \in [-180°, -150°] \bigcup [150°, 180°] \\ \text{front-right} & \angle s_2 s_1 \in [80°, 110°] \\ \text{front-left} & \angle s_2 s_1 \in [-110°, -80°] \\ \text{back} & \angle s_2 s_1 \in [-180°, -150°] \bigcup [150°, 180°] \\ \text{back-right} & \angle s_2 s_1 \in [80°, 110°] \\ \text{back-left} & \angle s_2 s_1 \in [-110°, -80°] \end{cases} \qquad (2)$$

Equation 2 reflects the symmetry of the angle measure used in our method, e.g., it can either represent front-right or back-left (based on the same angle) because the heading direction of a reference street is usually not indicated in sketch maps (so the referent street segment can be connected either to the end or to the start junction of the reference street segment). All the six relations are defined as cone-shape sectors with different sizes of ranges. The size of each range is generalized from our statistical survey of angles extracted from sketch maps [34]: among all the angles of connected streets from sketch maps, 98% of them represent either *straight* ($180° \pm 5°$) or *right* angles ($90° \pm 5°$) in metric maps. The reason to relax the definitions of both right and straight angles lies in the fact that participants can hardly draw the exact $90°$ and $180°$ angles. The probability for straight angles to fall into the interval $(150°, 180°)$ and the probability for right angles to fall into the interval $(80°, 110°)$ are both high (probability density function $P_{(X)} > .90$ with X being an interval of angles)[1]. Such statistical results are used to define the {front, back} relations to represent straight angles as well as the {front-right, front-left, back-right, back-left} relations to represent right angles. Figure 3 presents the six directional relations (left) and provides an example of measuring orientation of street segments in a sketch map (right). The orientation relations of street segments s_1, s_2, s_3 and s_4 with respect to the reference street segment (dotted-line) are front-left, front, back-right and back, respectively. The definition (Figure 3 left) also gives a complete picture of the angles measured from sketch maps. We found in our empirical data that angles falling into the interval $(70°, 120°)$ all represented right angles but with different density probabilities of smaller sub-intervals ($P_{(70°, 80°)} > .07, P_{(80°, 110°)} > .90, P_{(110°, 120°)} > .06$). The probabilities of the intervals $(70°, 80°)$ and $(110°, 120°)$ were quite low ($< .10$) so we could interpret that as people seldom drew right angles in the $(70°, 80°)$ or $(110°, 120°)$ interval.
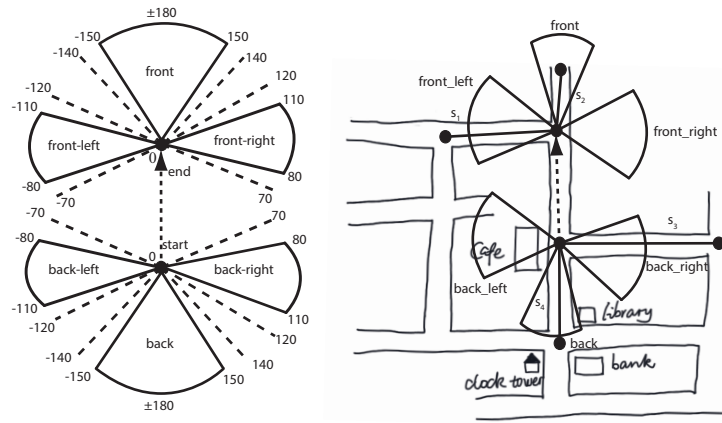

Figure 3: Definition (upper left) and example (right) of analyzing orientation of connected street segments.

Sketch aspect 3. *Orientation of landmarks with respect to a street segment* describes directional relations of landmarks with respect to their adjacent street segment. It is computed

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[1]This was done by the HDPInterval function provided by R to create a highest density.

at local level, i.e., only between adjacent landmarks and streets. The adjacency of a landmark l and a street segment s holds true if the distance between them is smaller than or equal to a threshold ε. Each landmark can have one or more adjacent street segments. Let $dist(l, s)$ be the function to calculate the distance between l and s, we define the adjacency of a landmark and a street segment in Equation 3:

$$Adjacency(l, s) = \begin{cases} \text{true} & dist(l, s) \leq \varepsilon \\ \text{false} & \text{otherwise} \end{cases} \tag{3}$$

Let street segment s be the reference object with its orientation pointing from its start junction to its end junction, s then determines two directional relations: $\{\text{left}, \text{right}\}$. Figure 4 shows how left and right are defined with respect to a reference street segment (left) and it provides an example (right). In the example (Figure 4 right), the orientation relations of landmark café and library with respect to their adjacent reference street segment are left and right, respectively.
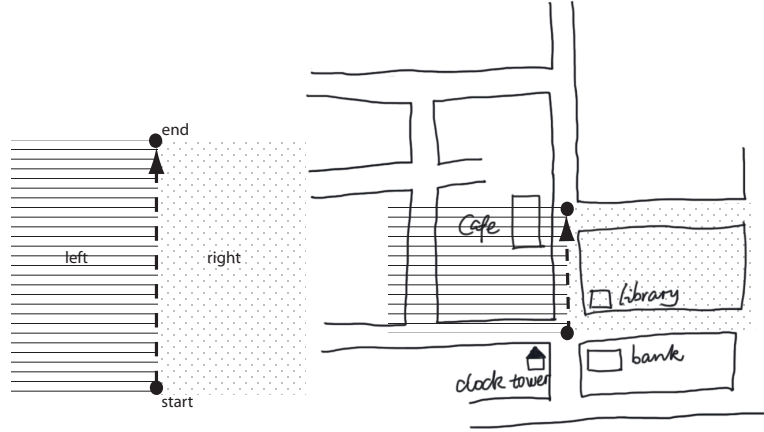

Figure 4: Definition (left) and an example of analyzing orientation of landmarks with respect to street segments (right).

Sketch aspect 4. *Cyclic order of street segments and landmarks around a junction* describes circular order relations of street segments and landmarks with respect to an adjacent reference junction. It is computed at local level, i.e., only between adjacent landmarks and junctions. The adjacency of a landmark and a junction holds true if the distance between them is smaller than or equal to a threshold ε. Each junction can have one or more adjacent landmarks. Let $dist(l, j)$ be the function to calculate the distance between a landmark l and a junction j, we can define the adjacency of a junction and a landmark in Equation 4:

$$Adjacency(l, j) = \begin{cases} \text{true} & dist(l, j) \leq \varepsilon \\ \text{false} & \text{otherwise} \end{cases} \tag{4}$$

Regarding types of referent objects available, the measure of cyclic order can include only street segments or both street segments and landmarks. Figure 5 shows an example

of measuring cyclic order with only street segments as referent objects (left) and another example with both street segments and landmarks as referent objects (right). The clockwise order of street segments with respect to junction j_1 is $(s_1, s_2, s_3, s_4, s_1)$ (Figure 5 left), and the clockwise order of both street segments and landmarks with respect to junction j_1 is $(s_1, s_2, l_1, s_3, l_2, s_4, l_3, s_1)$ (Figure 5 right). Note that the landmark café is not adjacent to the referent junction j_1 so it is excluded from cyclic order measure.
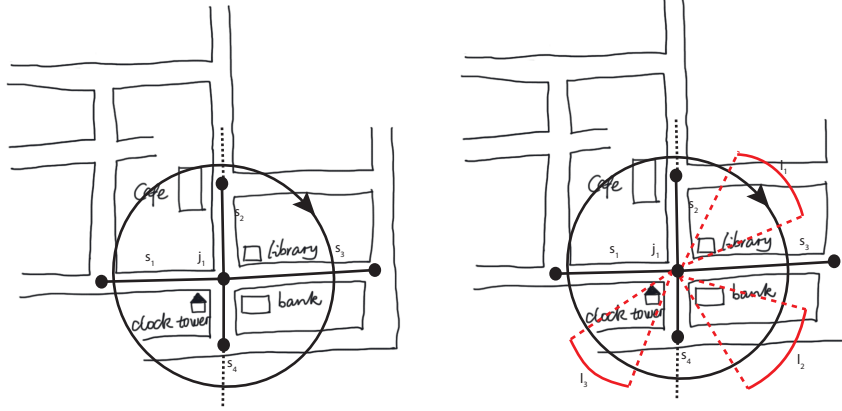

Figure 5: Examples of measuring cyclic order of street segments only (left) and both street segments and landmarks (right): l_1, l_2 and l_3 represent library, bank and clock tower, respectively.

Sketch aspect 5. *Linear order of street segments and landmarks along a route* describes the linear order relations of adjacent landmarks and street segments with respect to a route (local level). Regarding the side of a route, the measure of linear order can be left-sided, right-sided, or both-sided. Regarding types of reference objects available, the measure can include only street segments, only landmarks, or both street segments and landmarks. The distinctions above make up nine different types of linear order relations.

A *route* is defined as an orientated line of multiple connected street segments and has its origin and destination. We form a route by selecting the connected street segments in-between an origin and a destination, which maximizes the amount of information, i.e., the selection maximizes the number of street segments connected to the route as well as the number of landmarks located along the route. Depending on the distribution of landmarks and street segments, one or more routes can be formed in one sketch map. Let route R be a sequence of connected street segments $R = (s_1, s_2, \ldots, s_n)$. The adjacency of a street segment $s'(s' \notin R)$ and a landmark l with respect to R is defined in Equation 5:

$$
\begin{aligned}
Adjacency(s', R) &= \begin{cases} \text{true} & \text{if } \exists s : s \in R \land TopologyS(s', s) = connected \\ \text{false} & \text{otherwise} \end{cases} \\
Adjacency(l, R) &= \begin{cases} \text{true} & dist(l, R) \leq \varepsilon \\ \text{false} & \text{otherwise} \end{cases}
\end{aligned}
\tag{5}
$$

We represent a sequence of junctions that connect adjacent street segments to a route as a sequence of points. We project footprints of adjacent landmarks to a route as a sequence of intervals (Figure 6). Based on the linear order of points and intervals (as a pair of points) on the route, we distinguish four linear order relations as {before, overlap, equal, after} (Table 3). We do not further distinguish more complicated or fine-grained relations because our earlier study has revealed that this is an appropriate granularity level for free-hand sketch maps [36].

Relation	Descriptions
X before Y	X before Y without sharing any common part (I/P)
	X before Y and the start point of X coincides with the end point of Y (I)
X overlap Y	X and Y partially overlap each other (I)
X equal Y	X and Y are the same point (P)
	X and Y are the same start and end points (I)
X after Y	X after Y without sharing any common parts (I/P)
	X after Y and the end point of X coincides with the start point of Y (I)

Note. X, Y can be landmarks or street segments and projected to a route as intervals (I) or points (P).

Table 3: Definitions of linear order relations.

Figure 6 (left) visualizes the definition of linear order relations and Figure 6 (right) provides an example of measuring linear order in a sketch map. In Figure 6 (left), the route (blue dotted-line) is defined by two connected oriented street segments as $R = (r_1, r_2)$. Interval (i_1, i_2) is the projection of the landmark on the route. Junction j_1 is the point where the side street segment S and the route R meet. The linear order relation between the landmark and the side street segment can then be defined by the sequence of the points forming interval (i_1, i_2) and the point j_1 along the route.

In Figure 6 (right), the both-sided linear ordering with respect to route R ($R = (r_1, r_2, r_3)$) is: *bank overlap $library_1$, $library_1$ before s_1, s_1 equal s_2, s_2 before $library_2$, $library_2$ before café, café overlap s_3*. Note that library is located at the corner and is adjacent to both r_1 and r_2, so we use $library_1$ and $library_2$ to distinguish its two different projections to the route ($library_1$ is the projection to r_1 and $library_2$ is the projection to r_2). Street segment s_1 and s_2 are connected to the route at the same junction so their linear order along the route is equal. To identify these two street segments, further spatial analysis is required, e.g., the cyclic order relation introduced before.

Sketch aspect 6. *Topological relations of landmarks and city blocks* describe topological relations of landmarks with respect to city blocks (global level). We define a city block as a minimal region formed by a sequence of connected street segments. We define two types of topological relations, namely, inside and outside (Figure 7 upper-left). Depending on drawing styles, people draw landmarks touching the boundary of a city block or not: some people used to draw buildings like connected blobs or boxes touching each other though in reality these buildings do not share any common wall; and some other people used to draw buildings like separated blobs or boxes that are close to each other without touching. As our previous study revealed that participants do not distinguish the "inside" relation from "touching from inside" [36], we do not further distinguish these relations. Figure 7
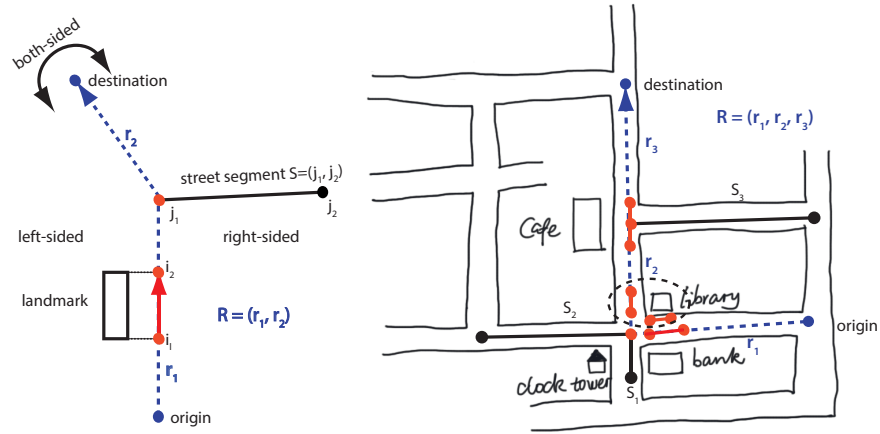
Figure 6: Definition of linear order relations (left) and an example of analyzing both-sided linear order relations (right). The two projections of library to route R are marked in circle.

(right) gives an example of analyzing topology of landmarks and city blocks in a sketch map: library is inside CB_1 but outside both CB_2 and CB_3, and café is inside CB_3 and outside CB_1 and CB_2.
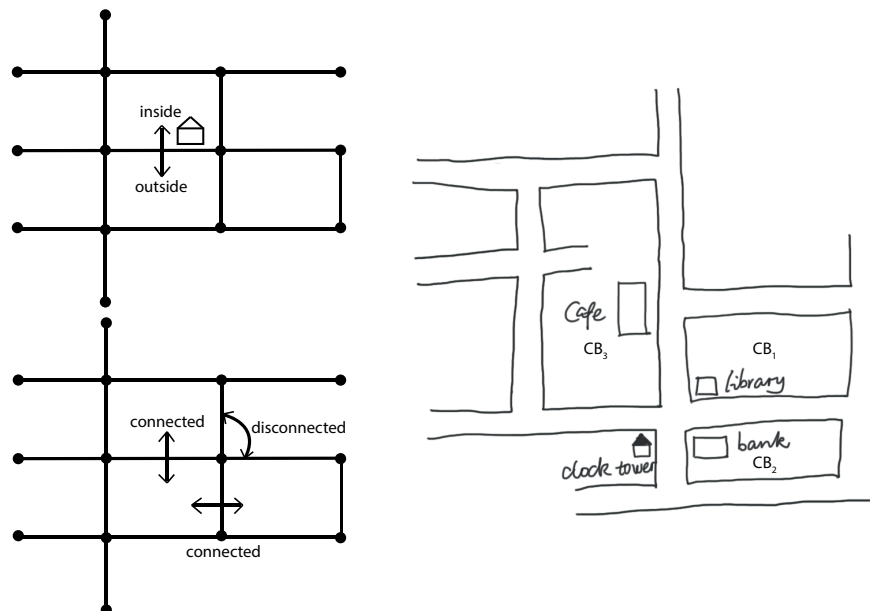
Figure 7: Definition (left) and an example of measuring topological relations of landmarks and city blocks (right).

Sketch aspect 7. *Topology of city blocks* describes the way in which constituent street segments forming city blocks are connected to each other (global level). Let city block CB be defined by a sequence of street segments $S = (s_1, s_2, \ldots, s_n)$ and let city block CB' be defined by another sequence of street segments $S' = (s'_1, s'_2, \ldots, s'_n)$ then two city blocks are connected, if they share at least one streets segment (Equation 6).

$$TopologyCB(CB, CB') = \begin{cases} connected & \text{if } \exists s : s \in S \wedge s \in S' \\ disconnected & \text{otherwise} \end{cases} \qquad (6)$$

Figure 7 provides a schematic view of defining the topology of three city blocks (lower-left) and gives an example of analyzing topology of city blocks in a sketch map (right). In the example (Figure 7 right), CB_1 is connected to both CB_2 and CB_3, and CB_3 is disconnected from CB_2 because they do not have any street segment in common.

4 Evaluation of accuracy and reliability

We carried out a human study to assess the proposed seven sketch aspects. Based on the results of this study, we confirmed the accuracy and reliability of the sketch aspects.

4.1 Drawing task and sketch map alignment

Participants. In total 21 participants with the average age of 26.5 years (sd $= 2.7$) took part in the study. Among these participants, 7 were females and 14 were males. Since people with cartographic knowledge might have an advantage in drawing maps, we balanced participants with and without geoscience background. Ten participants had a geosciences background and the remaining 11 did not. All the participants knew the study areas they chose to draw: they either visited the areas on a daily or weekly basis or they lived or worked in our study areas. We required our participants to be familiar with study areas, otherwise people would draw sketch maps with very little information, which makes it difficult to extract enough valid spatial information for spatial analysis.

Study areas. There were three study areas SA_1, SA_2 and SA_3. All of them were urban environments with a variety of natural and human-made objects and they were located in city Münster in Germany. The sizes of the study areas were all at the *environmental scale*, which requires locomotion to learn the environment and build cognitive maps [22]. We chose these study areas because they were homogeneous in that they had similar sizes, land cover and land use, numbers of landmarks, and structures of street networks (typical European non-grid like streets). We assumed that the invariant spatial information extracted from the sketch maps of the chosen study areas should be similar.

SA_1 (approximately 0.3km^2) was the area where our participants worked and/or lived. Thus, participants visited this area almost on a daily basis. SA_1 was characterized by built-up areas, which mostly were residential areas along both sides of a six-lane main street. The lake Aa, one of the top Münster attractions, was located at the north-western border of SA_1. There was no systematic street structure.

SA_2 (approximately 0.33km^2) was also an urban area, but buildings there were used for many different purposes. The study area contained a castle and its star-shape footprint

moat, a graveyard, a university cafeteria, and other university and residential buildings. Lake Aa was again the border of SA_2. Similar to SA_1, SA_2 did not have systematic grid-like street patterns.

SA_3 (approximately 0.18km^2) was the downtown of Münster encircled by a prominent cycling path. As the commercial and tourist heart of the city, SA_3 was the area with the highest density of shops, banks, restaurants, churches, museums, cafés, and bars. SA_3 was constructed with the Münster Cathedral as its center and other streets encircling the cathedral. The majority of movement in SA_3 relied on walking, which was different from other study areas allowing multiple transportations.

Procedure. The study had three parts. First, we provided participants with brief textual descriptions of the three study areas. The description of each study area included main streets, key landmarks, and spatial features to demarcate the boundary of each study area. Participants were asked to choose two familiar study areas and draw them only based on their memories. We provided participants with DIN-A4 size copy film with a base paper sheet covered with a transparent cover glued on the long side. Participants were asked to write down labels and annotations on the transparent cover and only make drawings on the base paper layer. By separating drawings from labels, we tried to avoid the effect that labels affect drawings and therefore spatial relations (e.g., people tend to draw longer streets if street names are longer and adjust footprints of landmarks to label sizes). In the second part of the experiment, we requested participants to fill in questionnaires to collect their background information including sketching skill, artistic talent, cartographic knowledge, spatial knowledge of the study areas, and the frequency of visit to the study areas. We wanted to test whether or not the background information had impact on sketch map accuracy. In the third part, we asked participants to align spatial objects in their sketch maps with corresponding objects in metric maps[2]. During alignment, participants tried to find correspondences between sketch maps and metric maps. (Sketch maps are usually highly aggregated and selective, therefore it is not always possible to identify alignments if you do not draw the map yourself.) The alignment results served as ground truth, which aided in the interpretation and analysis of what were recollected and drawn by participants. Having the ground truth, we could compare sketch and metric maps and then determine whether or not they have the same spatial relations for each sketch aspect (which was how we calculated sketch map accuracy).

4.2 Results and discussion

Participants drew in total 45 sketch maps. Three sketch maps were discarded because they were too simple to form a map structure. Among 42 valid sketch maps, 12 sketch maps depicted study area SA_1, 17 sketch maps depicted study area SA_2, and 13 sketch maps depicted study area SA_3. There were in total 490 landmarks, 1108 street segments, 589 junctions, and 164 city blocks extracted from valid sketch maps and included in the further evaluation. As displayed in Table 4, sketch maps vary similarly in the number of spatial objects included in individual maps. Particularly, the spread of street segments and junctions is more scattered than landmarks. Figure 8 shows two valid sketch maps drawn by the same participant.

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[2]The metric maps were snipped from the thematic layer of the official city map of Münster. http://www.muenster.de/stadt/katasteramt/geoinformationen.html

Study area	Basic measure	Individual sketch map			Total
		M	SD	Range	
SA_1	# of landmarks	10.08	3.26	6–17	121
	# of street segments	28.08	15.05	12–55	337
	# of junctions	15.33	9.42	4–32	184
	# of city blocks	4.33	4.25	0–13	52
SA_2	# of landmarks	11.41	4.81	6–23	194
	# of street segments	27.88	16.93	7–74	474
	# of junctions	14.71	10.15	3–42	250
	# of city blocks	4.29	4.21	0–15	73
SA_3	# of landmarks	13.46	5.47	7–24	175
	# of street segments	22.85	9.71	4–43	297
	# of junctions	11.92	5.07	4–21	155
	# of city blocks	3	2.65	0–7	39

Table 4: Distribution of spatial objects in sketch maps.

We applied the classification scheme proposed by Appleyard [1] to determine sketch map style based on complexity: sketch maps that predominately use sequential elements such as streets were defined as *sequential maps* whereas sketch maps that predominately use spatial elements such as individual shops and houses were defined as *spatial maps*; within each map type, according to map complexity ranging from primitive to complex, sequential maps can be further distinguished as fragmented, chain, branch and loop, and netted type, and spatial maps can be further distinguished as scattered, mosaic, linked, and pattern types. The majority of our sketch maps were of the most complex sequential-netted type (66.67% for SA_1, 70.59% for SA_2 and 69.23% for SA_3) (Table 5). Only two sketch maps were classified as spatial-linked type. The dominance of sequential maps (in total 95.2%) confirms the importance of the role that street network (junctions and street segments) plays in structuring sketch maps of urban areas. Sketch maps in Figure 8 are the examples of the sequential-netted type.

Study Area	Sketch map style	Frequency	Cell percentage	Cumulative percentage
SA_1	Sequential-chain	2	16.67	16.67
	Sequential-branch & loop	2	16.67	33.33
	Sequential-netted	8	66.67	100
SA_2	Sequential-chain	4	23.53	23.53
	Sequential-netted	12	70.59	94.12
	Spatial-linked	1	5.88	100
SA_3	Sequential-chain	2	15.38	15.38
	Sequential-branch& loop	1	7.69	23.08
	Sequential-netted	9	69.23	92.31
	Spatial-linked	1	7.69	100

Table 5: Frequency and percentage of sketch map style.

We evaluated both accuracy and reliability of proposed sketch aspects. For each sketch aspect, *accuracy* was calculated as the ratio of the number of correctly represented spatial relation divided by the total number of that spatial relation. The calculation of accuracy was

Sketch map of study area SA₁

Sketch map of study area SA₂

Figure 8: Two valid sketch maps from the same participant.

based on the ground truth provided by participants during the previous alignment session. *Reliability* was used to measure the extent to which an accurate sketch aspect yielded the same result in repeated conditions of same participants and homogeneous study areas. We asked participants to produce sketch maps of different study areas with similar settings and then examined if there existed any sketch aspect that was represented consistently accurate by different participants and across different study areas.

Table 6 gives an overview of the accuracy of proposed sketch aspects from individual study areas as well as the combined dataset including all study areas. Except for sketch aspect 3 (87.4% for SA_3) and sketch aspect 6 (77.6% for SA_3, 89.3% for the combined dataset)

all the other aspects show high accuracy rate ($> 90\%$) within individual study areas as well as in the combined dataset. The reason for the low accuracy of sketch aspect 6 is because one participant mirrored her sketches when she drew SA_3. The accuracy would be higher than 90% if we considered this sketch map an outlier.

Sketch aspect		Accuracy			
		SA_1	SA_2	SA_3	Combined
Topology of street segments Sketch aspect 1		99.5 183/184	99.2 248/250	94.2 146/155	98.0 577/589
Orientation of street segments Sketch aspect 2		98.5 332/337	97.0 460/474	97.3 289/297	97.7 1082/1108
Orientation of landmarks Sketch aspect 3		99.2 120/121	99.0 192/194	87.4 153/175	94.9 465/490
Cyclic order of street segments & landmarks Sketch aspect 4		100 197/197	99.3 141/142	96.8 179/185	98.7 517/524
Linear order of street segments & landmarks along route Sketch aspect 5	Left-sided 5l	99.5 203/204	99.1 232/234	92.5 99/107	98.0 534/545
	Right-sided 5r	99.0 200/202	98.8 249/252	95.3 182/191	97.8 631/645
	Both-sided 5b	97.8 398/407	98.2 478/487	90.1 272/302	96.0 1148/1196
Topology of landmarks & city blocks Sketch aspect 6		98.2 54/55	91.3 84/92	77.6 45/58	89.3 183/205
Topology of city blocks Sketch aspect 7		100 52/52	100 73/73	97.4 39/39	99.4 163/164

Table 6: Accuracy of proposed sketch aspects (% and absolute number of correct relations out of all relations).

The following four boxplots in Figure 9 display the dispersions of accuracy of sketch aspects. In Figure 9, the red dotted line ($y = 90$) denotes a 90% accuracy rate. Study areas SA_1, SA_2, and the combined dataset have most sketch aspects with their median values equal to 100 except for sketch aspect 5 (both-sided) and sketch aspect 2. Study area SA_3 has relatively more scattered distribution with more sketch aspects having larger IQR values that are contributed by the mirrored sketches we mentioned before. Nevertheless, there are still seven out of nine extended sketch aspects in SA_3 having their median values equal to 100, which implies the same central tendency as the other datasets. As a result, we can argue that all the evaluated sketch aspects show a high accuracy rate.

We also conducted a one-tailed one-sample t-test to further calculate the population mean of the sketch aspect accuracy based on the accuracy samples of SA_1, SA_2, SA_3, and the combined dataset.

We set the null and the alternative hypothesis as below:

H_0: The true mean of sketch aspect accuracy is equal to or lower than 90.
H_A: The true mean of sketch aspect accuracy is greater than 90.

The significance level of 0.01 was used. Table 7 presents the result of the one-tailed t-test. Based on the t-test result, we can argue that the proposed set of seven sketch aspects is able to yield the high percentage accuracy rate (> 90) thus it is accurate and reliable.
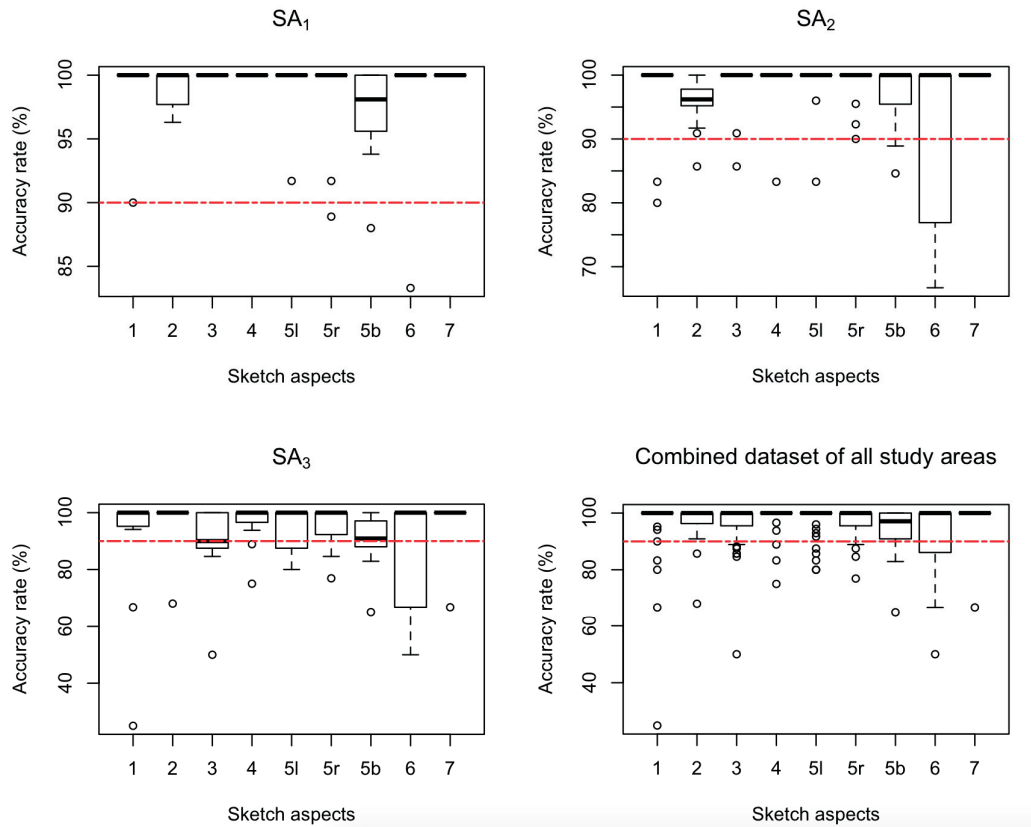
Figure 9: Boxplots of the accuracy distribution of individual and combined datasets (red-dotted line $y = 90\%$).

Sample dataset	Df	Estimated Mean	99% Cl
SA_1	105	99.03*	$[-\inf, 98.38]$
SA_2	141	97.42*	$[-\inf, 96.21]$
SA_3	116	93.08*	$[-\inf, 90.34]$
Combined	366	96.45*	$[-\inf, 95.40]$

Note. df = degree of freedom; Cl = confidence interval; *$p < .01$.

Table 7: One-tailed t-test of sketch aspect accuracy.

We analyzed participants' questionnaires, and we did not find that sketch map accuracy was correlated to participants' sketching skill, artistic talent, cartographic knowledge, and occupation. We also did not find any significant gender difference. The ten participants with geoscience background did not perform better than non-geo ones, i.e., sketch maps from geoscience participants were not more accurate than sketch maps drawn by non-geo participants. Some of the participants with geoscience background added carto-

graphic elements such as north arrow and legend to their sketches, but these elements did not contribute to sketch map accuracy. In fact, none of sketched north arrows pointed to the real north. All the participants visited their sketched areas by frequent active travel such as walking and cycling, and there was no significant difference found in sketch map accuracy from these two travel modes. The familiarity (quantified as frequency of visit to study areas) was the one that had impact on both sketch map accuracy and sketch map completeness: participants who visited sketched areas on a daily basis drew more buildings and more complete street networks with higher accuracy than those who visited on a monthly basis.

5   Conclusions and future work

Research in cognitive science and psychology applied sketch maps to analyze how cognitive processes lead to distortions and schematizations in cognitive map. There is a large body of literature on spatial information that is typically distorted in sketch maps. This paper provides another view on sketch maps where they can be well used as (reliable) data source for gathering information from humans if we focus on invariant spatial information. In the current paper, we identify sketched qualitative spatial information that is not distorted or schematized and does conform with what humans are able to recall and draw. We propose seven sketch aspects that preserve invariant spatial information and introduce a new spatial analysis method to represent and analyze these aspects. We evaluated accuracy and reliability of proposed sketch aspects by a human-subject experiment. In the experiment, we collected 42 free-hand sketch maps of three urban areas with 490 landmarks, 1108 street segments, and 589 junctions. All the sketch maps were drew by participants who actively visit sketched areas on a daily or weekly or at least monthly basis. The evaluation results could show that spatial relations captured by the proposed sketch aspects were hardly ever distorted or schematized. The accuracy of these sketch aspects was higher than 90% of all cases and such accuracy was also statistically significant. Our experiment also demonstrated the cognitive adequacy of the proposed aspects, i.e., the choice of sketch aspects was justified by the spatial information that people were able to recall and draw accurately from their memory. The paper shows that there is cognitively adequate, accurate, and reliable spatial information besides the distortions and schematizations explored by psychologists and cognitive scientists. Our study helps to understand sketch maps better in: (1) the types of spatial relations identified in sketch map; (2) the distinctions and measure of these spatial relations as a set of sketch aspects; and (3) the accuracy and reliability of sketch maps.

Moreover, the proposed seven sketch aspects provide the cognitive basis for formal representation and reasoning by suggesting invariant spatial information with appropriate distinctions of spatial relations and spatial reference frames. For more details on the formal representation of each sketch aspect, we refer to [26]. Corresponding qualitative representations can then be chosen to formally represent these sketch aspects [6, 17]. For instance, orientation of street segments (sketch aspect 2) suggests a formal calculus that uses local reference frame with distinctions of six relative relations instead of a global reference frame with cardinal directions. Such translation to a formal format that can be automatically calculated in a computational environment is essential to achieve a sketching interface for the project such as Sketchmapia [26]. The alignment of sketch and metric maps is essential for

developing sketching interface. Our follow-up study has shown successful alignment on some of the fundamental sketch aspects proposed in the current paper [5,7].

Future work will explore interrelations, dependency, dominance, and redundancy of sketch aspects. We may gain more knowledge of the ranking of sketch aspects based on dependency. Such a ranking will provide an insight to what to compute first in the actual process of sketch map alignment.

The way forward in studying invariant spatial information is the diversification of sketch maps from different perspectives, e.g., route sketch maps instead of survey sketch maps used in our study [35]; sketched areas visited by using different transport modes (active travel modes by foot, by bike, by car, or passive travel modes by bus and by taxi); different sketching spaces like indoor spaces or spaces with different spatial scales (e.g., city-size or even bigger); sketch maps with different locations such as urban areas with grid-like street networks or rural areas; and sketch maps with diverse map producers with different cultural backgrounds. Besides, there will be more choices of ontology and granularity used to define and represent spatial objects and their spatial relations in sketch maps.

Acknowledgments

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